

pressure; Q , volume flow rate of gas in bed (filtration rate); Q_0 , parameter in (7); q , total mass of gas entering per unit time; R , gas constant; S , cross-sectional area of apparatus; T , temperature; T_1, T_2 , lengths of static and dynamic periods of autooscillation cycle; t , time; V , volume of space under plate; w , velocity of piston; z , coordinate of lower surface of piston; α, β, γ , constants in (11); ν , frequency of autooscillations; ρ , density of gas; ρ' effective density of bed; σ , parameter in (7); φ , hydraulic resistance per unit volume; ω , parameter in (14). Indices: 1, 0, s, m, initial state of the bed in the static stage, the state of minimum quasifluidization, the stationary state, and the state with the maximum pressure drop, respectively; o, state of the gas above the bed.

LITERATURE CITED

1. A. A. Drinkenburg, in: Proceedings of the International Symposium on Fluidization, Netherlands University Press, Amsterdam (1967).
2. T. B. Anderson and R. Jackson, *Ind. Eng. Chem. Fund.*, 8, 137 (1969).
3. J. Verloop, P. M. Heertjes, and L. A. Lerk, *Chem. Eng. Sci.*, 29, 1109 (1974).
4. J. W. Hiby, in: Proceedings of the International Symposium on Fluidization, Netherlands University, Amsterdam (1967).
5. J. Verloop and P. M. Heertjes, *Chem. Eng. Sci.*, 29, 1035 (1974).
6. G. P. Cherepanov, *Ind. Eng. Chem. Fund.*, 11, 16 (1972).
7. Yu. A. Buevich and G. A. Minaev, *Inzh.-Fiz. Zh.*, 28, No. 15 (1975).
8. J. M. Matsen, S. Hovmand, and J. F. Davidson, *Chem. Eng. Sci.*, 24, 1743 (1969).
9. J. R. Grace, L. S. Krochmalnek, R. Clift, and E. Farkas, *Chem. Eng. Sci.*, 26, 617 (1971).
10. J. Baeyens and D. Geldart, *Chem. Eng. Sci.*, 29, 255 (1974).
11. Yu. A. Buevich, *Zh. Prikl. Mekh. Tekh. Fiz.*, No. 5, 155 (1967).
12. H. W. Wong and M. H. Baird, *Chem. Eng. J.*, 2, 104 (1971).
13. M. H. Baird and A. J. Klein, *Chem. Eng. Sci.*, 28, 1039 (1973).
14. J. F. Davidson, in: Tripartite Chemical Engineering Conference Symposium on Fluidization, Montreal (1968).

COOLING OF A COARSE LUMP IN A BED OF "FINE" PARTICLES

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The cooling of a coarse lump having the form of a rectangular prism in a blow-through bed of "fine" particles is discussed. A solution is obtained for large and small values of Fo by using Laplace transforms.

The heating or cooling of a polydisperse bed of lumps is a frequently occurring practical problem. Large lumps have many through pores so that a gaseous medium not only flows around a coarse lump, but also filters through its pores and increases the heat transfer.

Thus, the physical problem is the following. A bed through which air filters contains a coarse lump at a certain depth. At zero time the whole bed, including the lump, is heated to the temperature t_0 and is cooled by air with a temperature T' at the inlet to the bed. It is required to find the time to cool the coarse lump to a given temperature.

The following assumptions and simplifications are made.

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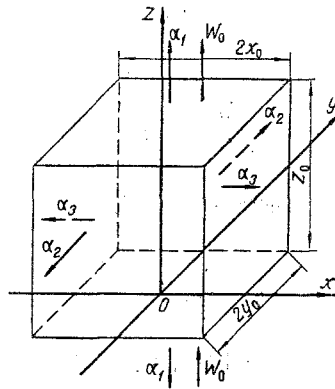


Fig. 1. Schematic diagram of the cooling of a coarse lump in a bed of "fine" particles.

1. The temperature of the gas everywhere near the coarse lump is equal to the temperature at the entrance to the lump. This temperature is determined by the dynamic cooling of a bed of "fine" particles whose diameter is found by averaging over the structure.

In certain practically important cases, for example, in the cooling of crushed sinter with lumps up to 200 mm in a bed of small lumps with an average diameter of 30-50 mm, the change in temperature of the gas over the height of a coarse lump occurs in an insignificant fraction of the cooling time so that in general it can be assumed to depend only on time. We consider such a case below.

2. The amount of air filtering through a lump is small, and the internal heat-transfer surface of a lump is large, so that without a large error the temperature of the gas in the pores can be taken equal to the temperature of the material.

3. The coarse lump is assumed to have the form of a rectangular parallelepiped with the gas flow at right angles to one of its faces.

A schematic diagram of the problem is shown in Fig. 1.

Assuming that the thermophysical properties of the bodies are constant the mathematical problem can be formulated as follows:

taking account of filtration the heat-conduction equation is

$$\frac{\partial \theta}{\partial Fo} = -M \frac{\partial \theta}{\partial Z} + K_X^2 \frac{\partial^2 \theta}{\partial X^2} + K_Y^2 \frac{\partial^2 \theta}{\partial Y^2} + \frac{\partial^2 \theta}{\partial Z^2}, \quad (1)$$

the boundary conditions are

$$Z = 0, \quad \frac{\partial \theta}{\partial Z} = Bi_0(\theta - \theta_0); \quad (2)$$

$$Z = 1, \quad \frac{\partial \theta}{\partial Z} = Bi_1(\theta - \theta_0); \quad (3)$$

$$Y = 0, \quad \frac{\partial \theta}{\partial Y} = 0; \quad (4)$$

$$Y = 1, \quad \frac{\partial \theta}{\partial Y} = Bi_2(\theta - \theta_0); \quad (5)$$

$$X = 0, \quad \frac{\partial \theta}{\partial X} = 0; \quad (6)$$

$$X = 1, \quad \frac{\partial \theta}{\partial X} = Bi_3(\theta - \theta_0); \quad (7)$$

the initial conditions are

$$Fo = 0; \quad \theta = 1. \quad (8)$$

The temperature of the gas θ depends on Fo . This dependence is determined from the known solution of the problem of cooling a bed of identical particles of average size [1].

We obtain the solution of the problem for a variable air temperature from the solution for constant temperature by using Duhamel's formula [2]. Denoting by ϑ' the solution of the problem for $\theta = 0$, we obtain

$$\vartheta = 1 - \int_0^{Fo} (1 - \theta(\varepsilon)) \frac{\partial \vartheta' (Fo - \varepsilon, X, Y, Z)}{\partial Fo} d\varepsilon. \quad (9)$$

We approximate $\theta(Fo)$ by a step function. Then

$$\vartheta = 1 - \sum_{i=1}^n (1 - \theta_i) \{ \vartheta' ((n-i)\Delta Fo, X, Y, Z) - \vartheta' ((n+1-i)\Delta Fo, X, Y, Z) \}. \quad (10)$$

Following [2] it is easy to show that the solution of Eq. (1) for $\theta = 0$ with boundary conditions (2)-(7) and initial conditions (8) can be written as the product of the solutions of the corresponding one-dimensional problems, i.e.,

$$\vartheta' = \vartheta'_1(Fo, Z) \vartheta'_2(Fo, Y) \vartheta'_3(Fo, X),$$

where ϑ'_1 satisfies the equation

$$\frac{\partial \vartheta'_1}{\partial Fo} = -M \frac{\partial \vartheta'_1}{\partial Z} + \frac{\partial^2 \vartheta'_1}{\partial Z^2} \quad (11)$$

and boundary conditions (2) and (3) ($\theta = 0$); ϑ'_2 and ϑ'_3 satisfy the one-dimensional heat-conduction equation whose solution for boundary conditions (4)-(7) is known [2].

Equation (11) with boundary conditions (2) and (3) and initial conditions (8) was solved for $\theta = 0$ by an operational method. By taking Laplace transforms we obtain ($s \rightarrow Fo$)

$$\begin{aligned} \bar{\vartheta}'_1 = & \frac{1}{s} + \frac{1}{\psi(s)} \left\{ Bi_1 \left[\left(\frac{M}{2} - Bi_0 \right) \operatorname{sh} \sqrt{\left(\frac{M}{2} \right)^2 + s} Z - \right. \right. \\ & - \left. \sqrt{\left(\frac{M}{2} \right)^2 + s} \operatorname{ch} \sqrt{\left(\frac{M}{2} \right)^2 + s} Z \right] \exp \left[\frac{M}{2} (Z-1) \right] - \\ & - Bi_0 \exp \left(\frac{M}{2} Z \right) \left[\left(\frac{M}{2} + Bi_1 \right) \operatorname{sh} \sqrt{\left(\frac{M}{2} \right)^2 + s} (1-Z) + \right. \\ & \left. \left. + \sqrt{\left(\frac{M}{2} \right)^2 + s} \operatorname{ch} \sqrt{\left(\frac{M}{2} \right)^2 + s} (1-Z) \right] \right\}, \quad (12) \end{aligned}$$

where

$$\begin{aligned} \psi(s) = & s \left\{ (Bi_1 + Bi_0) \sqrt{\left(\frac{M}{2} \right)^2 + s} \operatorname{ch} \sqrt{\left(\frac{M}{2} \right)^2 + s} - \right. \\ & \left. - \left[\frac{M}{2} (Bi_1 - Bi_0) - Bi_1 Bi_0 - s \right] \operatorname{sh} \sqrt{\left(\frac{M}{2} \right)^2 + s} \right\}. \quad (13) \end{aligned}$$

The numerator and denominator of Eq. (12) can be written as generalized polynomials by dividing by $\sqrt{(M/2)^2 + s}$. Consequently, the original can be found by using the expansion theorem [2].

After appropriate transformations we obtain

$$\begin{aligned} \vartheta'_1 = & \sum_{n=1}^{\infty} \frac{1}{\psi_n} \left\{ Bi_1 \exp \left[-\frac{M}{2} (1-Z) \right] \left[\mu_n \cos \mu_n Z - \right. \right. \\ & - \left. \left(\frac{M}{2} - Bi_0 \right) \sin \mu_n Z \right] + Bi_0 \exp \left(\frac{M}{2} Z \right) \left[\left(\frac{M}{2} + Bi_1 \right) \times \right. \\ & \left. \times \sin \mu_n (1-Z) + \mu_n \cos \mu_n (1-Z) \right] \left. \right\} \exp \left\{ \left[-\left(\frac{M}{2} \right)^2 - \mu_n^2 \right] Fo \right\}, \quad (14) \end{aligned}$$

where

$$\begin{aligned} \psi_n = & \left[\mu_n^2 + \left(\frac{M}{2} \right)^2 \right] \left\{ \frac{\sin \mu_n}{2} (Bi_1 + Bi_0 + 2) - \frac{\cos \mu_n}{2\mu_n} \times \right. \\ & \left. \times [Bi_1 + Bi_0 + Bi_1 Bi_0 - \mu_n^2 - \left(\frac{M}{2} \right)^2 - \frac{M}{2} (Bi_1 - Bi_0)] \right\}, \end{aligned} \quad (15)$$

μ_n is a root of the equation

$$\operatorname{ctg} \mu = \frac{\frac{M}{2} (Bi_1 - Bi_0) + \left(\frac{M}{2} \right)^2 - Bi_1 Bi_0 + \mu^2}{\mu (Bi_1 + Bi_0)}. \quad (16)$$

For $M = 0$, Eqs. (14)-(16) have a known solution [3]. The infinite series in (14) converges very slowly for large values of M and small Fo , and high accuracy is required in the calculation of the roots μ_n and the terms of the series.

Series (14) was summed on a Minsk-22 computer using double-length numbers [5] which increases the accuracy of representing numbers to 10^{-16} . It turned out that for $M \sim 20$ and any $Fo \geq 0.001$ it was sufficient to maintain a computational accuracy of $\sim 10^{-8}$ and 50 terms of the series. This accuracy was also sufficient for $M \sim 60$ and $Fo \geq 0.01$. For the range $0.01 > Fo \geq 0.003$ the accuracy had to be increased to 10^{-15} (50 terms of the series). Finally, for $Fo = 0.001$ even this accuracy and practically an infinite number of terms of the series (up to 200) were insufficient to obtain a concrete result, indicating the necessity for a further increase in accuracy. For large M ($M \sim 100$) even for $Fo = 0.003$ a result could not be obtained from (14) by using double-length numbers.

For small values of Fo (large s) the transform of the temperature can be written in the form

$$\bar{\theta}_i \approx \frac{1}{s} \frac{Bi_1 \exp \left[-\frac{M}{2} (1-Z) \right] \exp \left[-\sqrt{\left(\frac{M}{2} \right)^2 + s} (1-Z) \right]}{s \left(Bi_1 + \sqrt{\left(\frac{M}{2} \right)^2 + s + \frac{M}{2}} \right)} - \frac{Bi_0 \exp \left(\frac{M}{2} Z \right) \exp \left[-\sqrt{\left(\frac{M}{2} \right)^2 + sZ} \right]}{s \left(Bi_0 + \sqrt{\left(\frac{M}{2} \right)^2 + s - \frac{M}{2}} \right)}. \quad (17)$$

Using the displacement theorem and the Laplace transform inversion formulas [4], we obtain, finally,

$$\begin{aligned} \theta_i \approx & 1 - Bi_1 \left\{ \frac{\exp[-(1-Z)M]}{2(Bi_1 + M)} \operatorname{erfc} \left[\frac{1-Z}{2\sqrt{Fo}} - \frac{M}{2} \sqrt{Fo} \right] + \frac{1}{2Bi_1} \operatorname{erfc} \left[\frac{1-Z}{2\sqrt{Fo}} + \frac{M}{2} \sqrt{Fo} \right] - \right. \\ & \left. - \frac{Bi_1 + \frac{M}{2}}{(Bi_1 + M)Bi_1} \exp[Bi_1(1-Z) + Bi_1(Bi_1 + M)Fo] \operatorname{erfc} \left[\frac{1-Z}{2\sqrt{Fo}} + \left(Bi_1 + \frac{M}{2} \right) \sqrt{Fo} \right] \right\} - \\ & - Bi_0 \left\{ \frac{\exp(MZ)}{2(Bi_0 - M)} \operatorname{erfc} \left[\frac{Z}{2\sqrt{Fo}} + \frac{M}{2} \sqrt{Fo} \right] + \frac{1}{2Bi_0} \operatorname{erfc} \left[\frac{Z}{2\sqrt{Fo}} - \frac{M}{2} \sqrt{Fo} \right] - \right. \\ & \left. - \frac{Bi_0 - \frac{M}{2}}{Bi_0(Bi_0 - M)} \exp[Bi_0 Z + Bi_0(Bi_0 - M)Fo] \operatorname{erfc} \left[\frac{Z}{2\sqrt{Fo}} + \left(Bi_0 - \frac{M}{2} \right) \sqrt{Fo} \right] \right\}. \end{aligned} \quad (18)$$

It is natural to expect that in using Eq. (18) the most stringent restrictions on the values of Fo will be at the points $0.5 < Z \leq 1$ and the critical value of Fo will decrease with increasing M . Calculations show that for $M < 100$, Eq. (18) gives satisfactory results for $Z = 1$ and $Fo \leq 0.01$, agreeing with the computer solution of (14).

For large M we can write

$$\sqrt{\left(\frac{M}{2} \right)^2 + s} \approx \frac{M}{2} + \frac{s}{M}. \quad (19)$$

Using (19) in the transforms (12) and (13) for the argument of the hyperbolic functions, we obtain for the original

$$\begin{aligned} \theta_i \approx & 1 - \frac{\exp[-M(1-Z)]}{Bi_1 + M} \left\{ \left(Bi_1 + \frac{M}{2} \right) - \frac{M}{2} \operatorname{erf} \left(\frac{M}{2} \sqrt{Fo - \frac{1-Z}{M}} \right) - \left(Bi_1 + \frac{M}{2} \right) \times \right. \\ & \times \exp \left[(Bi_1^2 + Bi_1 M) \left(Fo - \frac{1-Z}{M} \right) \right] \operatorname{erfc} \left[\left(Bi_1 + \frac{M}{2} \right) \sqrt{Fo - \frac{1-Z}{M}} \right] \right\} \sigma \left(Fo - \frac{1-Z}{M} \right) - \frac{\sigma \left(Fo - \frac{Z}{M} \right)}{Bi_0 - M} \times \end{aligned}$$

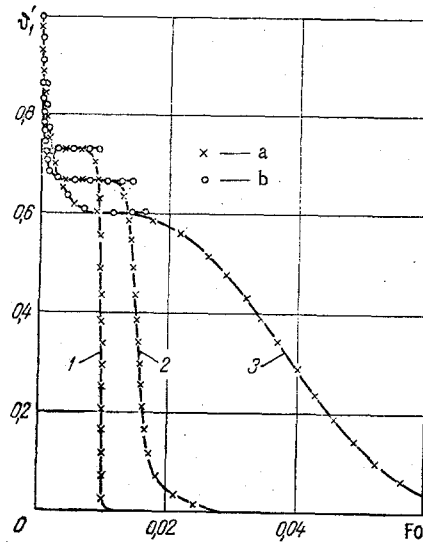


Fig. 2. Dependence of dimensionless temperature ϑ_1' on dimensionless time Fo for $Z = 1$: 1) $M = 104$; 2) 62.4 ; 3) 20.8 . a) Using (14); b) (18).

$$\begin{aligned}
 & \times \left\{ \left(Bi_0 - \frac{M}{2} \right) - \frac{M}{2} \operatorname{erfc} \left(\frac{M}{2} \sqrt{Fo - \frac{Z}{M}} \right) - Bi_0 - \frac{M}{2} \exp \left[(Bi_0^2 - Bi_0 M) \left(Fo - \frac{Z}{M} \right) \right] \right\} \times \\
 & \times \operatorname{erfc} \left[\left(Bi_0 - \frac{M}{2} \right) \sqrt{Fo - \frac{Z}{M}} \right] + Bi_0^2 Bi_1 \exp[-M(1-Z)] \int_0^{Fo - \frac{2-Z}{M}} \exp \left[- \left(\frac{M}{2} \right)^2 \varepsilon \right] \left\{ 2 \left[\left(Bi_1 + \frac{M}{2} \right)^2 \varepsilon + 1 \right] \times \right. \\
 & \times \sqrt{\frac{\varepsilon}{\pi}} - \left(Bi_1 + \frac{M}{2} \right) \varepsilon \exp \left[\left(Bi_1 + \frac{M}{2} \right)^2 \varepsilon \right] \left[2 \left(Bi_1 + \frac{M}{2} \right)^2 \varepsilon + 3 \right] \operatorname{erfc} \left[\left(Bi_1 + \frac{M}{2} \right) \sqrt{\varepsilon} \right] \left. \right\} d\varepsilon - \\
 & - \exp[-M(1-Z)] Bi_0 \left\{ 2 \left[\left(Bi_1 + \frac{M}{2} \right)^2 \left(Fo - \frac{2-Z}{M} \right) + 1 \right] \exp \left[- \left(\frac{M}{2} \right)^2 \left(Fo - \frac{2-Z}{M} \right) \right] \times \right. \\
 & \times \sqrt{\frac{Fo - \frac{2-Z}{M}}{\pi}} - \left(Bi_1 + \frac{M}{2} \right) \left(Fo - \frac{2-Z}{M} \right) \exp \left[(Bi_1^2 + Bi_1 M) \left(Fo - \frac{2-Z}{M} \right) \right] \left. \right\} \times \\
 & \times \left[2 \left(Bi_1 + \frac{M}{2} \right)^2 \left(Fo - \frac{2-Z}{M} \right) + 3 \right] \operatorname{erfc} \left[\left(Bi_1 + \frac{M}{2} \right) \sqrt{Fo - \frac{2-Z}{M}} \right]. \quad (20)
 \end{aligned}$$

As $M \rightarrow \infty$, solution (20) goes over into the following:

$$\vartheta_1' = \begin{cases} 1 & \text{for } Fo \leq \frac{Z}{M}; \\ 0 & \text{for } Fo > \frac{Z}{M}. \end{cases} \quad (21)$$

Some results for $Z = 1$ and various M are shown in Fig. 2, which compares results calculated by using (14) and (18).

NOTATION

$\vartheta(Fo, Z, Y, X) = (t - T')/(t_0 - T')$, temperature of lump; $\theta(Fo) = (T - T')/(t_0 - T')$, temperature of gas flowing around lump; $Z = z/z_0$, $X = x/x_0$, $Y = y/y_0$, coordinates of point; $Fo = \alpha t/z_0^2$, Fourier number; $K_X = z_0/x_0$, $K_Y = z_0/y_0$, geometrical characteristics of lump; $M = c_g \rho_g W_0 z_0 / \lambda$, criterion characterizing heat transfer by gas filtering through lump with velocity W_0 ; $Bi_0 = Bi_1 + M$, $Bi_1 = \alpha_1 z_0 / \lambda$, $Bi_2 = \alpha_2 y_0 / \lambda$, $Bi_3 = \alpha_3 x_0 / \lambda$, Biot numbers; $\alpha_1, \alpha_2, \alpha_3$, heat-transfer coefficients at corresponding faces; $\lambda, a = \lambda/c\rho, c, \rho$, thermal conductivity, thermal diffusivity, specific heat and apparent density of lump; c_g, ρ_g , specific heat and density of gas.

LITERATURE CITED

1. B. I. Kitaev, V. N. Timofeev, et al., Heat and Mass Transfer in a Packed Bed [in Russian], Metallurgiya, Moscow (1972), p. 219.
2. A. V. Lykov, Theory of Heat Conduction [in Russian], Visshaya Shkola, Moscow (1967).
3. B. F. Glikman, Zh. Tekh. Fiz., 27, No. 12 (1957).
4. V. A. Ditkin and A. P. Prudnikov, Handbook of Operational Calculus [in Russian], Visshaya Shkola, Moscow (1965).
5. Mathematical Provisions of the Minsk-22 Computer [in Russian], No. 4, Inst. Mat. Akad. Nauk, BelorusSSR, Minsk (1968).

THERMAL DISSOCIATION OF A POLYDISPERSE LUMP MATERIAL

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The problem of the dissociation of a polydisperse lump material is examined using the statistical approach and making due allowance for temperature-dependence of the coefficient of thermal conductivity.

A number of papers [1-5] have been written on the subject of the process of thermal dissociation of materials in lump form. It has been shown experimentally [2] that there is a fairly clear interface between the dissociated and undissociated substance which runs deep into the lump. This interface is the surface at which the heat passing through the shell of reacted substance is consumed. The model of a heat exchanger with a variable heat-exchange surface [5] is, therefore, suitable for use as a physical model of the process which is compatible with the experiment.

A formula for the time required for the dissociation of a single lump under the conditions of constant thermophysical process characteristics is devised in [5] on the basis of several physically sound hypotheses which simplify the investigation. It is, however, a well-known fact [6-8] that the thermophysical characteristics of substances being heated are not constant, in particular, the coefficient of thermal conductivity can be described as a linear function of temperature:

$$\lambda = \lambda_0(1 + \omega t). \quad (1)$$

It should be noted that if this relationship is disregarded for industrial furnace operating conditions, there will be significant errors in the calculation of the material dissociation time. In addition, when samples are heated, their porosity p is changed according to the relation [9, 10]

$$p = 115.2 - 0.078 t. \quad (2)$$

By definition

$$p = \frac{\rho_l - \rho_a}{\rho_l} 100. \quad (3)$$

The dependence of the coefficient of thermal conductivity on the temperature and apparent volumetric mass of the material has been found in [7] in the form

$$\lambda = 1.163(-1.011 - 0.066 \cdot 10^{-2} t + 1.513 \cdot 10^{-3} \rho_a). \quad (4)$$

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